Advances in Optical Metrology --- 14 June 2016

Ultimate bounds for quantum and Sub-Rayleigh imaging

Cosmo Lupo & Stefano Pirandola

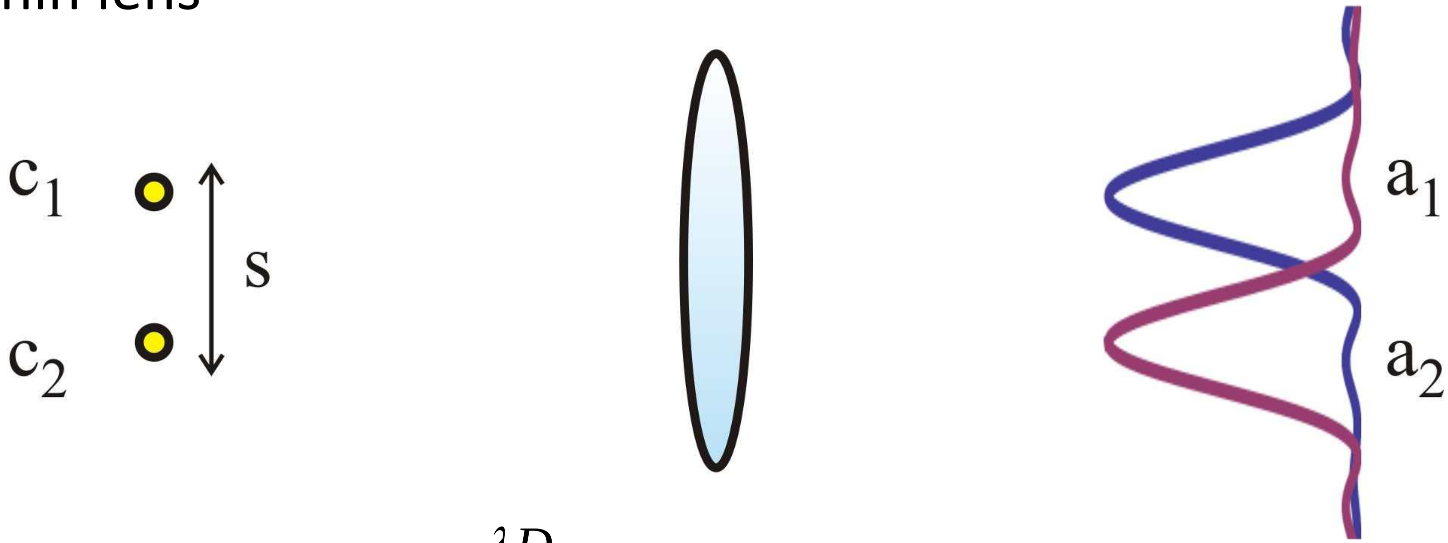
University of York

arXiv:1604.07367

Introduction

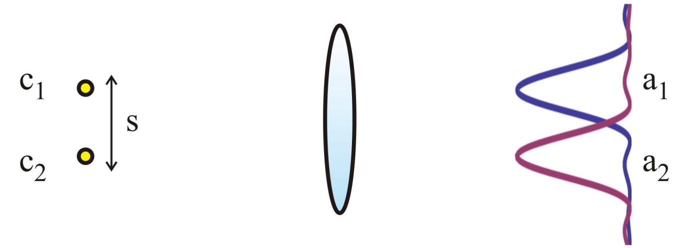
Quantum imaging for metrology

- Linear
- Diffraction-limited
- Paraxial approximation
- Far field
- Thin lens



Rayleigh length $x_R = \frac{\lambda D}{R}$

Introduction
Quantum imaging for metrology



Communication theory: 1
how much information can be transmitted through the optical system?

Hypothesis testing: 2
is the light emitted by one source or two?

Parameter estimation: 3
how much is the distance between the two sources?

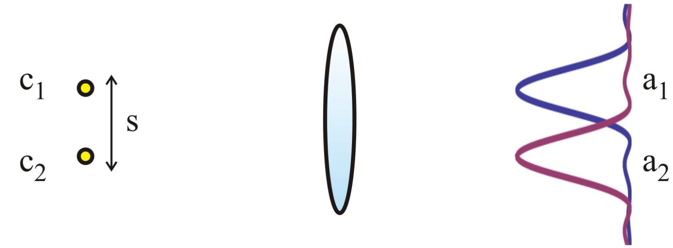
Tsang, Nair, Lu
arXiv: 1511.00552

Cramer-Rao bound $$\Delta s \geq \frac{1}{\sqrt{\mathrm{QFI}_s}}$$

Quantum Fisher information QFI_s

Introduction

Quantum imaging for metrology



What are the optimal sources?

what happen for non-classical or entangled light?

How much can we resolve sub-Rayleigh features?

what is the ultimate bound for sub-wavelength imaging?

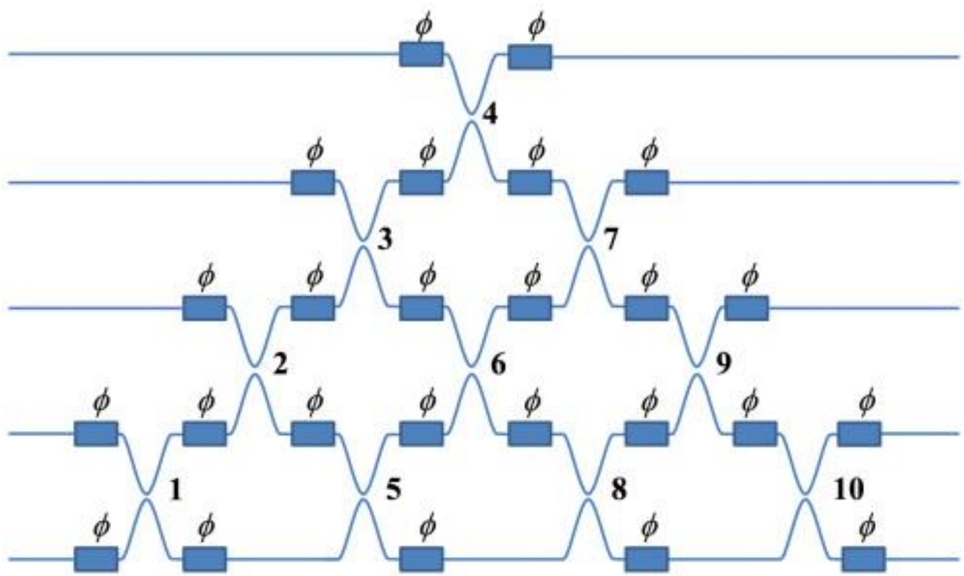
How is the scaling of Δs with the number of photons?

Shot noise $\Delta s \sim \frac{1}{\sqrt{N}}$ vs Heisenberg $\Delta s \sim \frac{1}{N}$

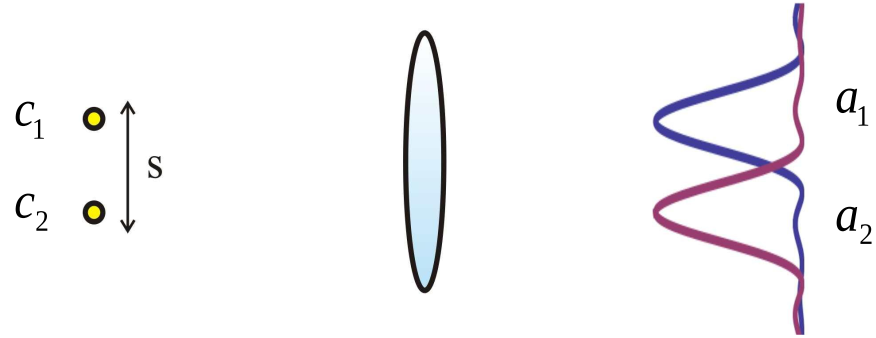
An intuition

Any linear optical imaging system is formally equivalent to a circuit made of beam splitter and phase shifter

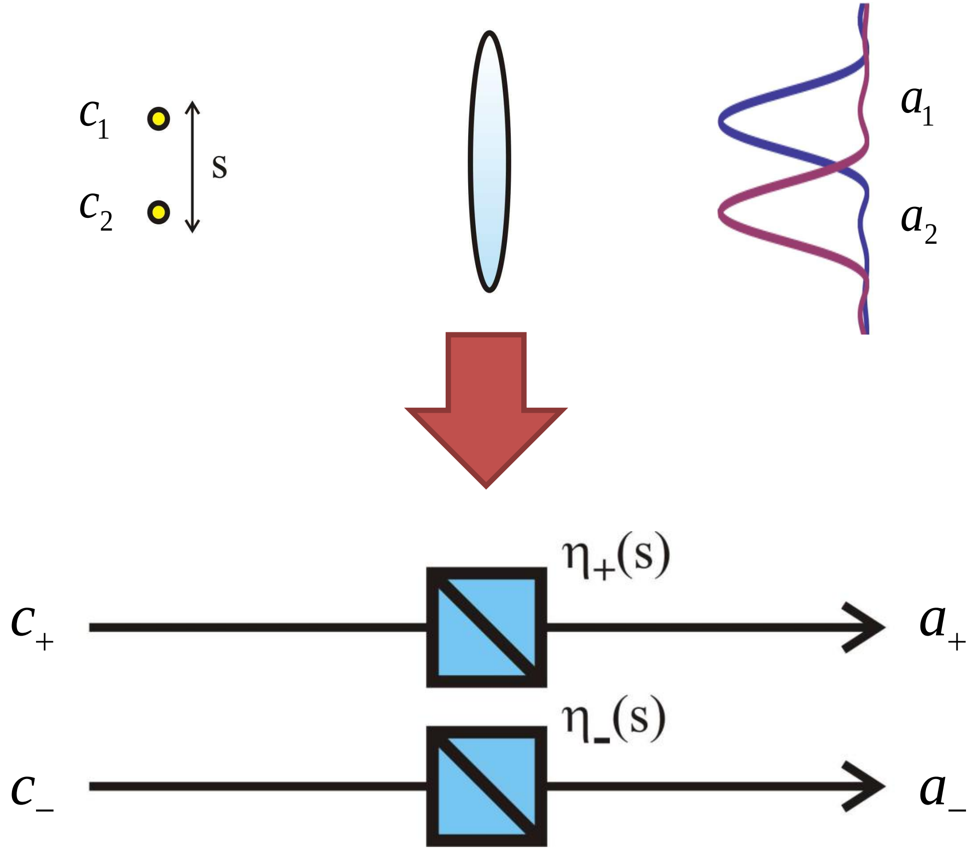
Reck, Zeilinger, Bernstein, Bertani, PRL 1994



Going abstract



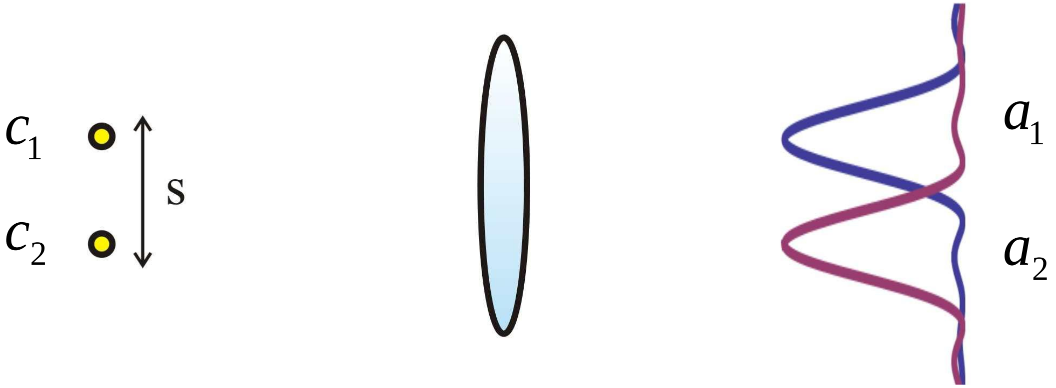
Going abstract



$$c_\pm = \frac{c_1 \pm c_2}{\sqrt{2}}$$

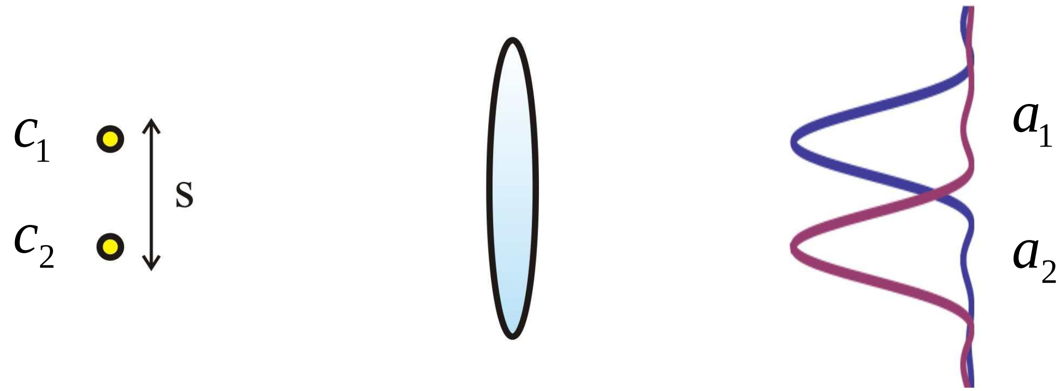
$$a_\pm = \frac{a_1 \pm a_2}{\sqrt{2(1 \pm \delta)}}$$

Dynamical equations



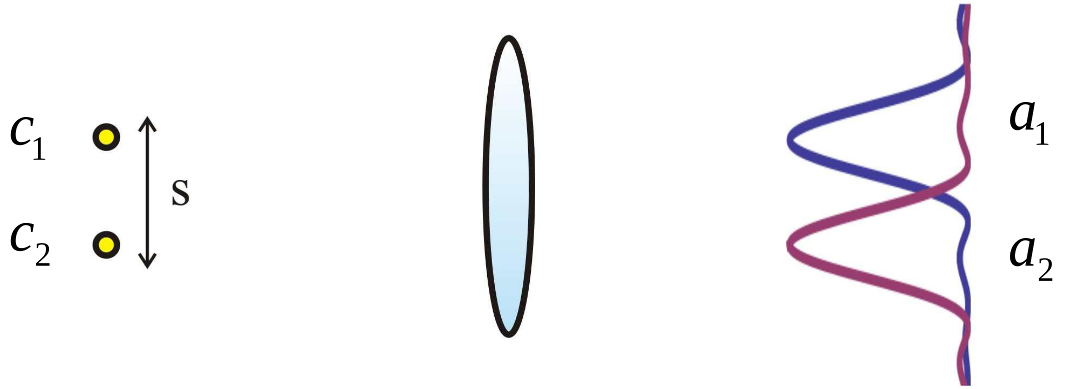
$$\frac{da_{\pm}}{ds}$$

Dynamical equations



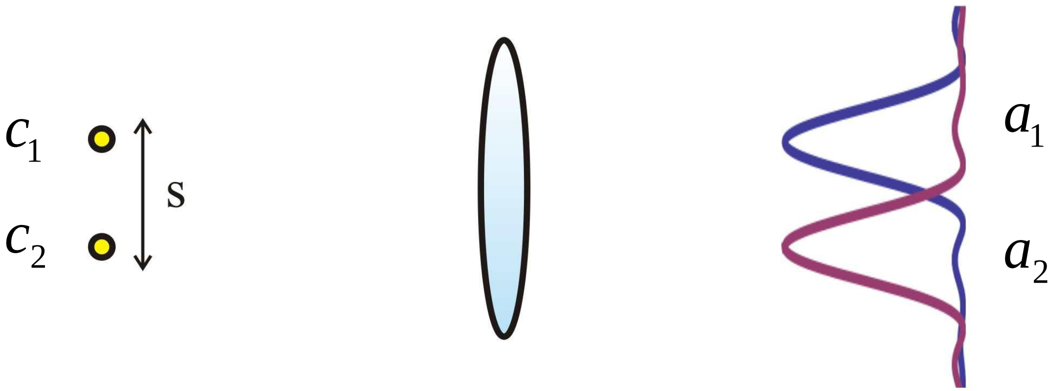
$$\frac{da_{\pm}}{ds} = i\left[H_{\pm}, a_{\pm}\right]$$

Dynamical equations



$$\frac{da_{\pm}}{ds} = i\left[H_{\pm}, a_{\pm}\right] + \frac{\partial a_{\pm}}{\partial s}$$

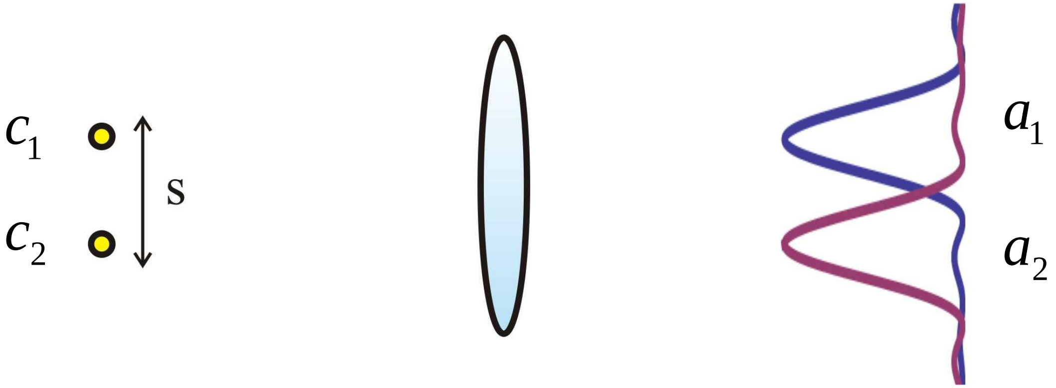
Dynamical equations



$$\frac{da_\pm}{ds} = i\left[H_\pm, a_\pm\right] + \frac{\partial a_\pm}{\partial s}$$

$$\frac{da_\pm}{ds} = i\left[H_\pm^{\text{eff}}, a_\pm\right]$$

Dynamical equations



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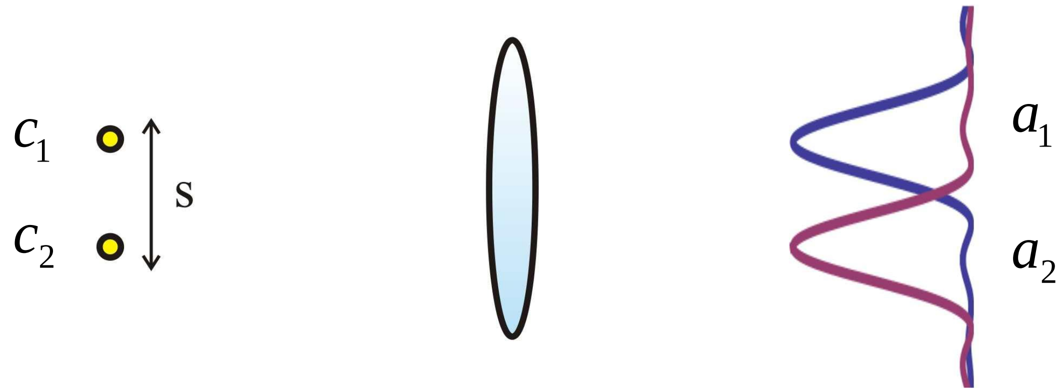
$$\frac{da_\pm}{ds} = i\left[H_\pm^{\text{eff}}, a_\pm\right]$$

$$H_\pm^{\text{eff}} = i\omega_\pm\left(a_\pm^\dagger b_\pm - a_\pm b_\pm^\dagger\right)$$

Dynamical equations



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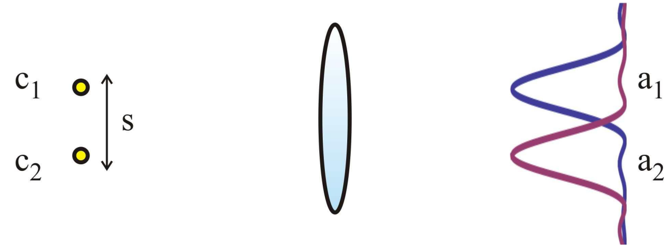
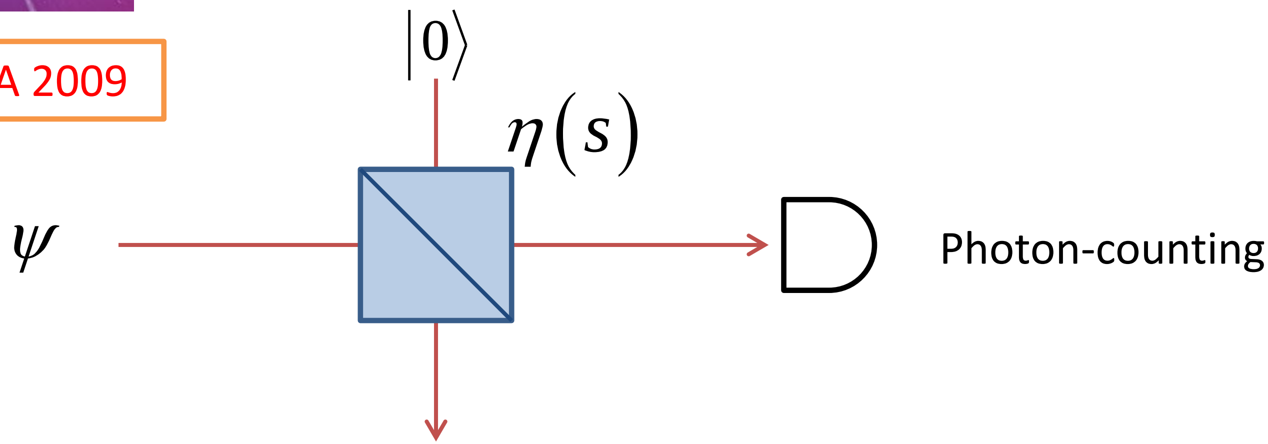
Beam-splitter Hamiltonian

Beam-splitter (review)



Adesso et al., PRA 2009



$$\psi = \left| N \right\rangle$$

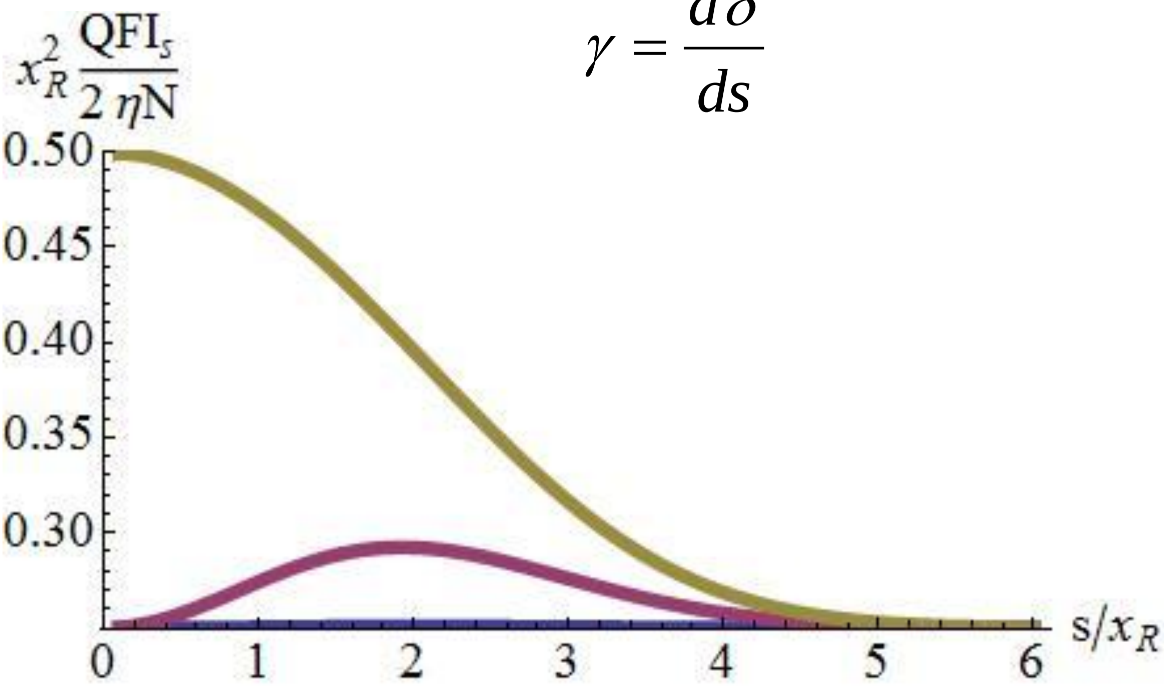
$$\mathrm{QFI}_{\eta} = \frac{N}{\eta(1-\eta)} \quad \Rightarrow \quad \mathrm{QFI}_{s}$$

Ultimate precision bound

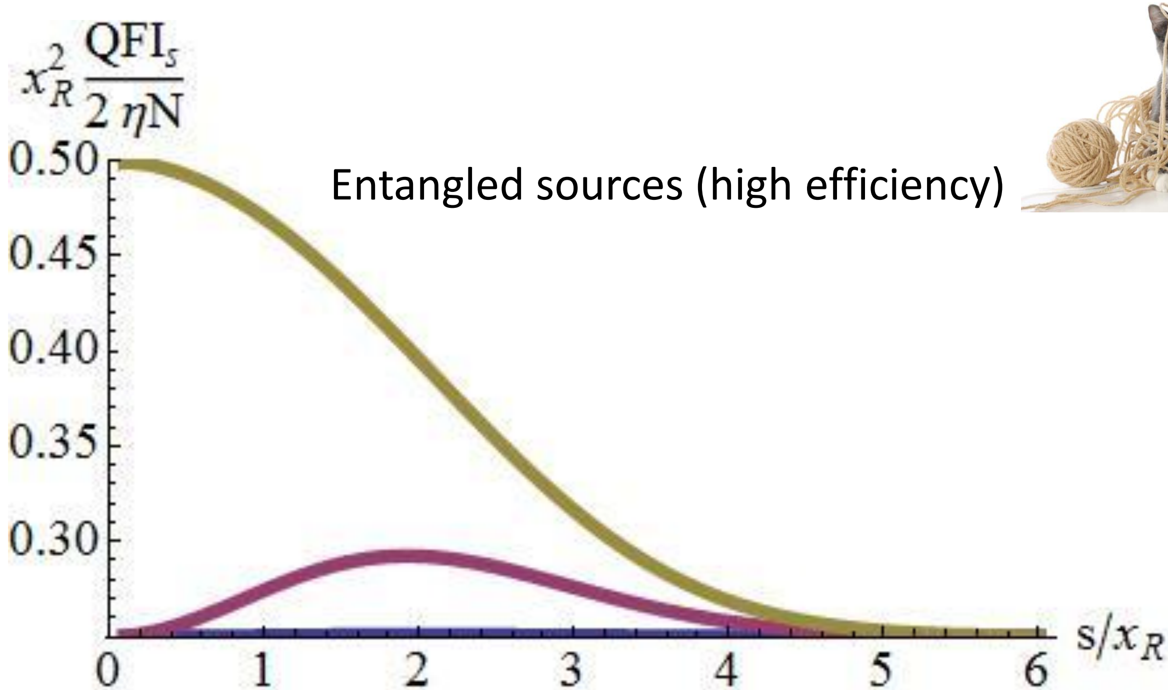
$$\max \mathrm{QFI}_s = 2\eta N\left(\frac{\eta(1-\eta)\gamma^2}{(1-\eta)^2-\delta^2\eta^2}+\frac{\alpha}{x_\mathrm{R}^2}\right)$$

$$\gamma = \frac{d\delta}{ds}$$

$$\Delta s \geq \frac{1}{\sqrt{\mathrm{QFI}_s}}$$



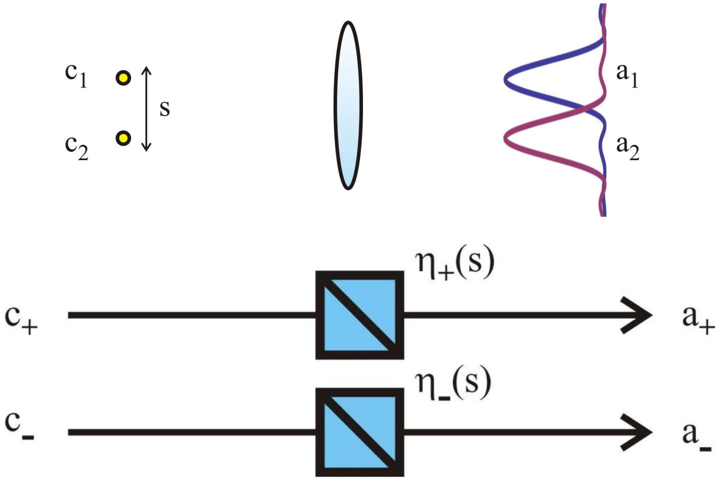
Optimal and almost-optimal sources

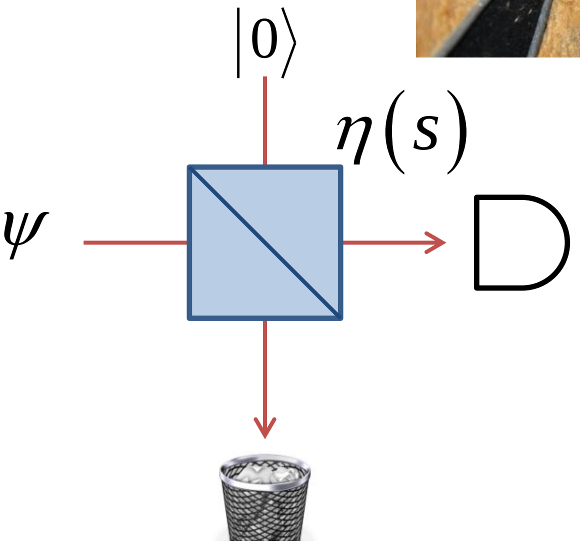


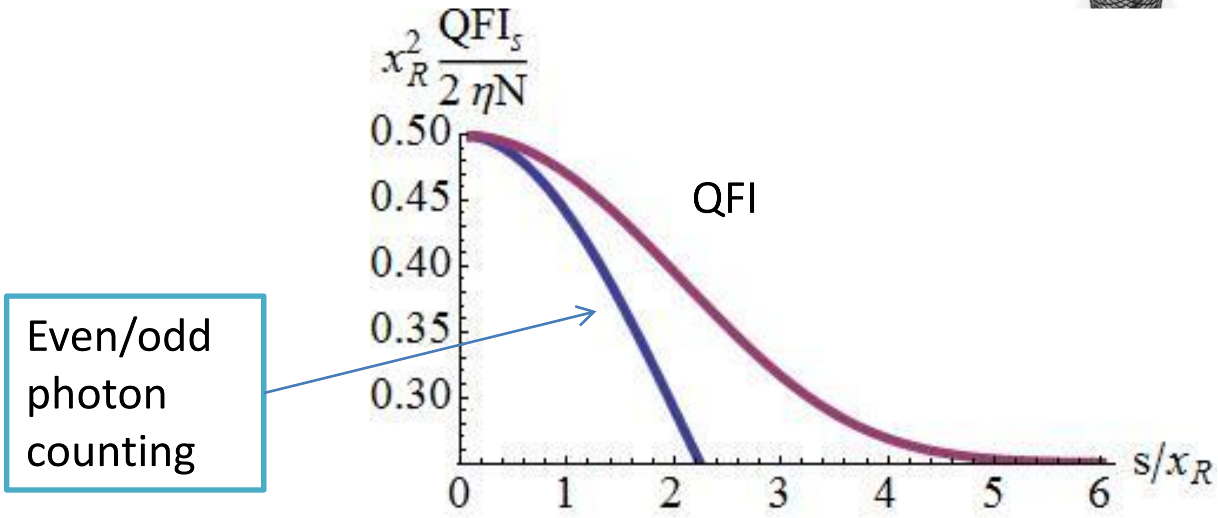
Entangled sources (high efficiency)

Thermal / incoherent sources (high attenuation)

Sub-optimal measurement







Conclusion

- Problem solved at a formal level
- Optimal measurements?
- Standard quantum limit (shot-noise scaling)
- Entangled sources are optimal (but only by a small factor)
- Thermal sources are practically optimal
- Non-linear imaging, non-paraxial